$$13x_1 + 13x_2 \geq 117$$
$$x_1 , x_2 \geq 0$$

$$3x_1 + 4x_2 \geq 12,$$
$$x_1 , x_2 \geq 0$$

have no feasible solution.

21. A furniture manufacturer wishes to determine the number of tables and chairs to be made by him in order to optimize the use of his available resources. These products utilize two different types of timber and he has on hand 1500 board feet of the first type and 1000 board feet of the second type. He has 800 man-hours available for the total job. Each table and chair requires 5 and 1 board feet respectively, of the first type of timber and 2 and 3 board feet respectively of the second type. 3 man-hours are required to make a table and 2 man-hours are needed to make a chair. The manufacturer makes a profit of Rs. 12.00 on a table and Rs. 5.00 on a chair. Write out the complete linear programming formulation of the problem in terms of maximizing the profit.

[**Ans.** Max. $z = 12x + 5y$, x = no. of tables, y = no. of chairs,

s.t. $5x + y \leq 1500$, $2x + 3y \leq 1000$, $3x + 2y \leq 800$, $x, y \geq 0$.]

22. A factory is engaged in manufacturing two products A and B which involve lathe work, grinding and assembling. The cutting, grinding and assembling times required for one unit of A are 2, 1 and 1 hours respectively and for one unit of B are 3, 1 and 3 hours respectively. The profits on each unit of A and B are Rs. 2.00 and Rs. 3.00 respectively.

Assuming that 300 hours of lathe time, 300 hours of grinding time and 240 hours of assembling time, are available, pose a linear programming problem in terms of maximizing the profit on the items manufactured.

[**Ans.** Max. $z = 2x_1 + 3x_2$, x_1 = no. of units of A, x_2 = no. of units of B,

s. t. $2x_1 + 3x_2 \leq 300$, $x_1 + x_2 \leq 300$, $x_1 + 3x_2 \leq 240$, $x_1 , x_2 \geq 0$.]

23. At a cattle breeding firm it is prescribed that the food ration for one animal must contain at least 14, 22 and 1 units of nutrients A, B and C respectively. Two different kinds of fodder are available. Each unit weight of these two contains the following amounts of the three nutrients :

	Fodder 1	Fodder 2
Nutrient *A*	2	1
Nutrient *B*	2	3
Nutrient *C*	1	1

It is given that the costs of unit quantity of fodder 1 and 2 are 3 and 2 monetary units respectively. Pose a linear programming problem in terms of minimizing the cost of purchasing the fodders for the above cattle breeding firm. [*Calcutta Hons., 1988*]

[**Ans.** Min. $z = 3x_1 + 2x_2$, x_1, x_2 (≥ 0) are the unit weights of the fodder 1 and fodder 2, s. t. $2x_1 + x_2 \geq 14$, $2x_1 + 3x_2 \geq 22$, $x_1 + x_2 \geq 1$.]

24. Three products are processed through three different operations. The times (in minutes) required per unit of each product, the daily capacity of the operations (in minutes per day) and the profit per unit sold for each product (in rupees) are as follows :

Operation	Time per unit			Operation capacity
	Product I	Product II	Product III	
1	3	4	3	42
2	5	0	3	45
3	3	6	2	41
Profit	3	2	1	

The zero time indicates that the product does not require the given operation. The problem is to determine the optimum daily production for three products that maximizes the profit.

Formulate the above production planning problem as a linear programming problem assuming that all units produced are sold.

[**Ans.** If x_1, x_2, x_3 be the number of units produced daily of products 1, 2 and 3, then the L. P. P. is

Max. $z = 3x_1 + 2x_2 + x_3$

s. t. $3x_1 + 4x_2 + 3x_3 \leq 42$, $5x_1 + 3x_3 \leq 45$,

$3x_1 + 6x_2 + 2x_3 \leq 41$, x_1, x_2, $x_3 \geq 0$.]

25. A firm, manufacturing two types of medicine A and B, can make a profit of Rs. 20 per bottle of A and Rs. 30 per bottle of B. Both A and B need for their production two essential chemicals C and D. Each bottle of A requires 3 litres of C and 2 litres of D and each bottle of B requires 2 litres of C and 4 litres of D. The total supply of these chemicals are 210 litres of C and 300 litres of D. The total supply of these chemicals are 210 litres of C and 300 litres of D. Type B medicine contains alcohol and its manufacture is restricted to 65 bottles per month. How many bottles each of A and B should the firm manufacture per month to maximize its profit of the products ? How much is this profit ? (Use graphical method).

[**Ans.** 30 bottles A type and 60 bottles B type. The profit is Rs. 2400.]

26. A chemical firm stores its output of glucose at depots in Delhi and Madras. The depot at Delhi has 100 quintals of glucose in store whereas that at Madras has 150 quintals. Two customers A and B have their works at Patna and Nagpur respectively and at a particular time require 120 quintals and 110 quintals respectively. The cost to the supplier of delivering one quintal is given in the following table :

	From Delhi	From Madras
To Patna	Rs. 50	Rs. 20
To Nagpur	Rs. 60	Rs. 30

The Delhi depot has arranged already for a sub-contractor to deliver 30 quintals out of 110 quintals required at Nagpur and this quantity cannot be reduced. Find how distribution should be organised to have the minimum delivery cost.

[**Ans.** If x quintals be delivered from Delhi to Patna, $(120 - x)$ quintals will be delivered from Madras to Patna, $(80 - x)$ quintals from Delhi to Nagpur and $(30 + x)$ quintals from Madras to Nagpur ; Min. cost = Rs. 8100.]

27. Old hens can be bought for Rs. 2.00 each but young one costs Rs. 5.00 each. The old hens lay 3 eggs per week and young ones 5 eggs per week, each egg being worth 30 paise. A hen costs Re. 1.00 per week to feed. If I have only Rs. 80.00 to spend for purchasing the hens, then how many of each kind should I buy to have a maximum profit per week assuming that I cannot house more than 20 hens ? (Pose the problem and solve graphically).

What additional constraint is to be added to the constraint set in order that the profit will be more than Rs. 6.00 per week ?

[**Ans.** 16 young hens only ; Max. profit = Rs. 8.00 ; $0.5x_2 - 0.1x_1 \geq 6$, if x_1 and x_2 be the number of old and young hens respectively.]

28. (*a*) An agricultural farm has 180 tons of Nitrogen fertilizers, 250 tons of phosphate and 220 tons of potash. It is able to sell 3 : 3 : 4 mixtures of these substances at a profit of Rs. 15 per ton and 1 : 2 : 1 mixtures at a profit of Rs. 12 per ton respectively. Pose a linear programming problem to show how many tons of these two mixtures should be prepared to obtain the maximum profit.

[**Ans.** Let x_1 and x_2 tons of these two mixtures be produced.

Max. $z = 15x_1 + 12x_2$ subject to $6x_1 + 5x_2 \le 3600$,

$3x_1 + 5x_2 \le 2500$, $8x_1 + 5x_2 \le 4400$ and $x_1, x_2 \ge 0$.]

(*b*) Three different types of lorries *A*, *B* and *C* have been used to transport 60 tons solid and 35 tons liquid substance. *A* type lorry can carry 7 tons solid and 3 tons liquid, *B* type lorry can carry 6 tons solid and 2 tons liquid, *C* type lorry can carry 3 tons solid and 4 tons liquid. The costs of transport are Rs. 500.00, Rs. 400.00 and Rs. 450.00 per lorry of *A*, *B* and *C* respectively. To find the minimum cost, formulate the problem mathematically. [*Calcutta Hons., 2006*]

[**Ans.** Min. $z = 500x_1 + 400x_2 + 450x_3$ subject to $7x_1 + 6x_2 + 3x_3 \ge 60$, $3x_1 + 2x_2 + 4x_3 \ge 35$ and $x_1, x_2, x_3 \ge 0$, if x_1 lorries of *A* type, x_2 lorries of *B* type and x_3 lorries of *C* type be used.]

29. Upon completing the construction of his house Mr. Sharma discovers that 100 sq. ft. of plywood scrap and 80 sq. ft. of white pine scrap are in usable form for the construction of tables and book-cases. It takes 16 sq. ft. of plywood and 8 sq. ft. of white pine to make a table ; 12 sq. ft. of plywood and 16 sq. ft. of white pine are required to construct a book-case. By selling the finished products to a local furniture store Mr. Sharma can realise a profit of Rs. 25.00 on each table and Rs. 20.00 on each book-case. How may he most profitably use the left over wood ? Use graphical method to solve the problem. [*Kalyani Hons., 1990*]

[**Ans.** 4 tables, 3 book-cases ; Max. profit = Rs. 160.00.]

30. A hospital has the following requirements for nurses :

Period	Clock time (24 hours day)	Minimum number of nurses required
1	6 A.M. — 10 A.M.	60
2	10 A.M. — 2 P.M.	70
3	2 P.M. — 6 P.M.	60
4	6 P.M. — 10 P.M.	50
5	10 P.M. — 2 A.M.	20
6	2 A.M. — 6 A.M.	30

Nurses report to the hospital wards at the beginning of each period and work for eight consecutive hours. The hospital wants to determine the minimum number of nurses so that there may be sufficient number of nurses available for each period. Formulate this as an *L. P. P.*

[*Calcutta Hons., 1983, 1997*]

[**Ans.** Min. $z = \Sigma x_j$ subject to $x_1 + x_6 \geq 60$, $x_2 + x_1 \geq 70$, $x_3 + x_2 \geq 60$, $x_4 + x_3 \geq 50$, $x_5 + x_4 \geq 20$, $x_6 + x_5 \geq 30$, $x_j \geq 0$, x_j is the number of nurses starting work at the beginning of the j-th period.]

31. A transport company has offices in five localities, *A, B, C, D* and *E*. Some day the offices located at *A* and *B* had 8 and 10 spare trucks whereas offices at *C, D, E* required 6, 8 and 4 trucks respectively. The distances in kilometres between the five localities are given below :

To		C	D	E
From	A	2	5	3
	B	4	2	7

How should the trucks from *A* and *B* be sent to *C, D* and *E* so that the total distance covered by the trucks is minimum. Pose the problem as an *L. P. P.* [*Calcutta Hons., 2004*]

[Let x trucks be sent from *A* to *C* and y trucks be sent from *A* to *D*. Then the number of trucks from *A* to *E* will be $\{8 - (x + y)\}$, that from *B* to *C* will be $(6 - x)$, that from *B* to *D* will be $(8 - y)$ and that from *B* to *E* will be $\{10 - (6 - x) - (8 - y)\}$. All these quantities and x, y are positive. The total distance covered, which is to be minimised, is

$$2x + 5y + 3(8 - x - y) + 4(6 - x) + 2(8 - y) + 7(x + y - 4).]$$

32. (*a*) A toy company manufactures two types of doll, a basic version-doll *A* and a deluxe version-doll *B*. Each doll of type *B* takes twice as long to produce as one of type *A* and the company would have time to make a maximum 2000 per day if it produces only the basic version. The supply of plastic is sufficient to produce 1500 dolls per day (both *A* and *B* combined). The deluxe version requires a fancy dress of which there are only 600 per day available. If the company makes a profit of Rs. 3 and Rs. 5 per doll respectively on doll *A* and *B*, then how many of each should be produced per day in order to maximize the profit ?

[**Ans.** *A* type 1000, *B* type 500 ; Max. profit = Rs. 5500.]

(*b*) A used car dealer wishes to stock-up his lot to maximize his profit. He can select cars *A*, *B* and *C* which are valued wholesale at Rs. 5000, Rs. 7000 and Rs. 8000 respectively. These can be sold at Rs. 6000, Rs. 8500 and Rs. 10500 respectively. For each car type the probability of sale are :

Type :	*A*	*B*	*C*
Probability of sale :	0.7	0.8	0.6

For every two cars of *B* he should buy one car of type *A* or *C*. He has Rs. 100000 to invest. Formulate a linear programming problem to maximize his expected gain.

[**Ans.** x_1, x_2, x_3 are the number of cars purchased of type *A*, *B* and *C*. The total expected gain $z = 1000\, x_1 \times 0.7 + 1500\, x_2 \times 0.8 + 2500\, x_3 \times 0.6$. Investment constraints are $x_1, x_2, x_3 \geq 0$, $5000 x_1 + 7000\,(2x_2) \leq 100000$ and $7000\,(2x_2) + 8000\, x_3 \leq 100000$.]

(*c*) A manufacturer produces three models I, II, III of a certain product. He uses two types of raw materials *A* and *B* of which 4000 and 6000 units respectively are available. The raw material requirements per unit of three models are given below :

Raw material	Requirement per unit of model		
	I	II	III
A	2	3	5
B	4	2	7

The labour time for each unit of model I is twice that of model II and three times that of model III. The entire labour force of the factory can produce the equivalent of 2500 units of model I. A market survey indicates that the minimum demand of the three models are 500, 500 and 375 units respectively. However the ratio of the number of units produced must be equal to 3 : 2 : 5. Assume that the profit per unit of models I, II and III are Rs. 60, Rs. 40 and Rs. 100 respectively. Formulate the problem as a linear programming model in order to determine the number of units of each product which will maximize his profit.

[*Vidyasagar Hons., 2003*]

[**Ans.** Max. $z = 60\, x_1 + 40\, x_2 + 100\, x_3$ subject to $2x_1 + 3x_2 + 5x_3 \leq 4000$, $4x_1 + 2x_2 + 7x_3 \leq 6000$, $x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 \leq 2500$, $\frac{1}{3}x_1 = \frac{1}{2}x_2$, $\frac{1}{2}x_2 = \frac{1}{3}x_3$ and $x_1 \geq 500$, $x_2 \geq 500$, $x_3 \geq 375$.]